today's lesson
<table>
<thead>
<tr>
<th><strong>scalar</strong></th>
<th><strong>vector</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>only magnitude, <strong>no direction</strong></td>
<td>both magnitude and direction</td>
</tr>
<tr>
<td><strong>1-dimensional measurement of quantity</strong></td>
<td><strong>not 1-dimensional</strong></td>
</tr>
<tr>
<td>time, mass, volume, speed temperature and so on</td>
<td>force, velocity, momentum, acceleration, and so on</td>
</tr>
<tr>
<td>scalar</td>
<td>vectors</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>time</td>
<td>lift</td>
</tr>
<tr>
<td>area</td>
<td>drag</td>
</tr>
<tr>
<td>mass</td>
<td>force</td>
</tr>
<tr>
<td>work</td>
<td>thrust</td>
</tr>
<tr>
<td>speed</td>
<td>weight</td>
</tr>
<tr>
<td>power</td>
<td>velocity</td>
</tr>
<tr>
<td>length</td>
<td>momentum</td>
</tr>
<tr>
<td>volume</td>
<td>acceleration</td>
</tr>
<tr>
<td>density</td>
<td>displacement</td>
</tr>
<tr>
<td>temperature</td>
<td>magnetic field</td>
</tr>
</tbody>
</table>
discussing now......

pseudovector vs. polar vector
pseudovector = axial vector
pseudovector

- under an improper rotation (reflection) gains additional sign flip

- in 3-dimension, equal magnitude, opposite direction

- Geometrically, not mirror image
pseudovector

not mirror image
polar vector = real vector
A polar vector in 2-dimension has magnitude and direction. It can also have length and angle. On reflection, it matches with its mirror image.
Polar Vector

- inherently possesses direction
- direction remains unchanged irrespective of the coordinate system chosen
- when its components are reversed, the vector obtained is different from the original vector
<table>
<thead>
<tr>
<th><strong>pseudovector (axial vector)</strong></th>
<th><strong>polar vector (real vector)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>transforms like a vector,</td>
<td>same</td>
</tr>
<tr>
<td>under <em>proper</em> rotation</td>
<td></td>
</tr>
<tr>
<td>gains additional sign flip,</td>
<td>does not</td>
</tr>
<tr>
<td>under <em>improper</em> rotation</td>
<td></td>
</tr>
<tr>
<td><em>not</em> mirror image</td>
<td>matches with its mirror image,</td>
</tr>
<tr>
<td>on reflection</td>
<td>on reflection</td>
</tr>
<tr>
<td><em>does not</em> reverse sign when</td>
<td>reverses sign when</td>
</tr>
<tr>
<td>the coordinate axes are</td>
<td>the coordinate axes are</td>
</tr>
<tr>
<td>reversed</td>
<td>reversed</td>
</tr>
<tr>
<td>common examples: magnetic</td>
<td>common examples: velocity,</td>
</tr>
<tr>
<td>field, angular velocity</td>
<td>force, momentum</td>
</tr>
</tbody>
</table>
In elementary mathematics, the representation of a vector involves specifying its length (size) and angle specifying its endpoints in polar coordinates.
polar coordinates = radial coordinate
angular coordinate = polar angle

they are defined in terms of Cartesian coordinates

\[ x = r \cos \theta \]
\[ y = r \sin \theta \]

\( r \) = radial distance from the origin
\( \theta \) = the counter-clockwise angle from the x-axis
Cartesian coordinate system
right-hand rule
the direction of a vector is represented as the counter-clockwise angle of rotation
easy measurement: magnitude and direction

12 km East

8 km North

vector
What is radius vector?

The radius vector \( \mathbf{r} \) = “position vector” from the origin to the current position

\[
\mathbf{r} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{r} \cdot \mathbf{v}
\]

\( \mathbf{v} = \text{velocity} \)
The radius for circular motion = radius vector
The radius vector locates the orbiting object
The radius vector is not constant
(constant size, direction changes)

O = origin at the center of the circle
vector cross product rule

\[ \text{pseudovector} \times \text{pseudovector} = \text{pseudovector} \]

\[ \text{polar vector} \times \text{polar vector} = \text{pseudovector} \]

\[ \text{polar vector} \times \text{pseudovector} = \text{polar vector} \]
vector triple product rule

polar vector \times [\text{polar vector} \times \text{polar vector}] = \text{polar vector}

\text{pseudovector} \times [\text{polar vector} \times \text{polar vector}] = \text{pseudovector}

polar vector \times [\text{pseudovector} \times \text{pseudovector}] = \text{polar vector}

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\text{pseudovector} \times [\text{polar vector} \times \text{pseudovector}] = \text{polar vector}

\text{polar vector} \times [\text{polar vector} \times \text{pseudovector}] = \text{pseudovector}
vector A under inversion

vector B under inversion

original vector A

original vector B

their cross product remains invariant
discussing now

Vector Geometry
What is the Geometrical representation of vectors?

directional arrow segments
equal vectors
same magnitude
same direction

negative of a vector
same magnitude
opposite direction
vector diagrams

“Tip & Tail rule”
vector diagrams are illustrated by 2 points

tip of the vector: destination of the arrow-head

tail of the vector: the point of origin
expressing vectors using components

A vector can be split into 2 orthogonal components by drawing an imaginary rectangle.
expressing vectors using components

\[ A_x = A \cos \theta \]

\[ A_y = A \sin \theta \]

\[ A = \sqrt{A_x^2 + A_y^2} \]

\[ \theta = \tan^{-1} \frac{A_y}{A_x} \]
addition of vectors & subtraction of vectors
addition of vectors
using “Tip & Tail rule”
addition of vectors using components

- Vector A
  - Vector components: $A_x$, $A_y$

- Vector B
  - Vector components: $B_x$, $B_y$
determining magnitude and direction of resultant vectors

Pythagorean theorem

Trigonometric functions
The Pythagorean Theorem

\[(\text{hypotenuse})^2 = (\text{opposite})^2 + (\text{adjacent})^2\]
addition of 2 orthogonal vectors using Pythagorus’ theorem

1. complete the right-handed triangle,
2. draw the hypotenuse $= A + B$
addition of 2 orthogonal vectors using Pythagorus’ theorem

1. complete the rectangle,
2. draw the diagonal = $A + B$
Pythagorean theorem applies to orthogonal vectors.
Pythagorean theorem is **NOT** applicable to

adding *more than two vectors*

vectors that are *not* at **90°** angle with each other
Trigonometric functions apply to non-orthogonal vectors.
rules of addition of vectors

Parallelogram rule

Trigonometric rule
addition of vectors

Parallelogram rule

vector sum \( C = A + B \)

scalar sum \( c \neq a + b \)
addition of vectors

Trigonometric rule

\[ \frac{A}{\sin \theta_1} = \frac{B}{\sin \theta_2} = \frac{C}{\sin \theta_3} \]

\[ C = \sqrt{A^2 + B^2 - 2AB \cos \theta_3} \]
subtraction of 2 vectors

vector A

vector B

vector sum C

construct the negative of vector B

construct the resultant C

C = A - B
subtraction of 2 vectors

A − B

vector A

−vector B

vector B
useful links

http://physics.info/
http://www.wikidiff.com/
http://study.com/academy/lesson/
http://phrontistery.info/index.html
http://physics.stackexchange.com/
http://physics.tutorvista.com/motion/
http://whatis.techtarget.com/definition/
http://hyperphysics.phy-astr.gsu.edu/hbase/
http://www.efm.leeds.ac.uk/CIVE/CIVE1140/section01/
http://www.physicsclassroom.com/class/1DKin/Lesson-1/
http://www.eolss.net/sample-chapters/c05/e6-10-03-00.pdf
http://hydrorocketeers.blogspot.dk/2011_06_01_archive.html
useful links

http://physics.weber.edu/

http://theory.uwinnipeg.ca/physics/

http://mathworld.wolfram.com/topics/

http://plato.stanford.edu/entries/equivME/

http://plato.stanford.edu/entries/qt-uncertainty/

http://www.physicsclassroom.com/class/circles/

http://physics.about.com/od/classicalmechanics/

https://www.aip.org/history/exhibits/heisenberg/p08.htm

http://www.begellhouse.com/books/4799b1c911961558.html

https://owlcation.com/stem/Physics-Definition-and-Branches


useful links

http://physics.info/photoelectric/

https://en.wikipedia.org/wiki/General_relativity

http://whatis.techtarget.com/definition/quantum-theory

http://www.space.com/17661-theory-general-relativity.html

http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/ltrans.html


http://www.physicsoftheuniverse.com/topics_relativity_general.html


http://ircamera.as.arizona.edu/NatSci102/NatSci102/lectures/physicallaws.htm
